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BLACK HOLES IN STRING THEORY

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Abstract: A black hole is a region of space time exhibiting such strong gravitational effects that nothing not even particles and electromagnetic radiation such as light can escape from inside it. From the theoretical point of view, black holes provide an intriguing arena in which to explore the challenges posed by the reconciliation of general relativity and quantum mechanics. Since string theory purports to provide a consistent quantum theory of gravity, it should be able to address these challenges. In fact, some of the most fascinating developments in string theory concern quantum-mechanical aspects of black hole physics. These are the subject of this paper. This paper is devoted to trying to find a microscopic quantum description of black holes. One of the most important achievements of string theory in recent times is the construction of examples that provide an affirmative the thermodynamic description of black holes. The thermodynamics of strings is governed largely by the exponential growth of the number of quantum states accessible to a string, as a function of its energy. The behavior of the entropy indicates that at high energies the temperature approaches a finite constant, the Hagedorn temperature. We explain how the counting of string states can be used to give a statistical mechanics derivation of the entropy of black holes. The calculations give results in qualitative agreement with the entropy of Schwarzschild black holes and in quantitative agreement with the entropy of certain charged black holes.
Key words: *String Theory, Black Hole, Entropy, D-branes.*

1. Introduction

In 1972, Bekenstein was the first to suggest that black holes should have a well-defined entropy. He wrote that a black hole's entropy was proportional to its (the black hole's) event horizon. Bekenstein also formulated the generalized second law of thermodynamics, black hole thermodynamics, for systems including black holes. Both contributions were affirmed when Stephen Hawking proposed the existence of Hawking radiation two years later. In 1974, Hawking performed a lengthy calculation that convinced him that particles do indeed emit from black holes. Today this is known as Bekenstein-Hawking radiation. In most physical systems the thermodynamic entropy has a statistical interpretation in terms of counting microscopic configurations with the same macroscopic properties, and in most cases this counting requires an understanding of the quantum degrees of freedom of the system. String theory, being a theory of quantum gravity, should be able to describe black holes. As we shall see, although string theory is usually well approximated by local quantum field theory, in the neighborhood of a black hole horizon the differences become extreme. The analysis of these differences suggests a resolution of the black hole dilemma and a completely new view of the relations between space, time, matter, and information.

1. Black holes in general relativity

In order to introduce the reader to some basic notions of black-hole physics, let us begin with the simplest black-hole solutions of general relativity in four dimensions, which are the Schwarzschild and Reissner -Nordström black holes. The latter black hole is a generalization of the Schwarzschild solution that is electrically charged. Another generalization, known as the Kerr black hole, is a black hole with angular momentum.

1.1 Schwarzschild black hole

In Schwarzschild coordinates, the Schwarzschild geometry is manifestly spherically symmetric and static. In Schwarzschild coordinates (t, r, θ, φ) the metric is given by:

$$ds^2 = -(1-2\frac{MG}{r})dt^2 - (1-2\frac{MG}{r})^{-1}dr^2 - r^2 d\Omega^2 = g_{\mu\nu}dx^u dx^v$$

where:

$$d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2.$$

The coordinate t is called Schwarzschild time, and it represents the time recorded by a standard clock at rest at spatial infinity, 2GM is known as the Schwarzschild radius and denoted r_H, G is Newton’s constant and the coordinate r is called the Schwarzschild radial coordinate. The Schwarzschild metric only depends on the total mass M and it reduces to the Minkowski metric as M → 0. The surface r =r_H, called the event horizon, separates the previous two regions. This metric is stationary in the sense that the metric components are independent of the Schwarzschild time coordinate t, so that $\partial/\partial t$ is a Killing vector. This Killing vector is time-like outside the horizon, null on the horizon, and space-like inside the horizon. It becomes clear that M has the interpretation of a mass by considering the weak field limit, that is, the asymptotic r → ∞. In this limit we should recover Newtonian gravity. The Newtonian potential ϕ in these stationary coordinates can be read off from the tt component of the metric:

$$g_{tt} : -(1+2\phi).$$

As a result, in the case of the Schwarzschild black hole,

$$\phi = -\frac{MG}{r}.$$

The parameter M is the black-hole mass.

The four-dimensional Schwarzschild metric can be generalized to D dimensions, where it takes the form:

$$ds^2 = -\left[1-(\frac{r_H}{r})^{D-3}\right]dt^2 + \left[1-(\frac{r_H}{r})^{D-3}\right]^{-1}dr^2 + r^2 d\Omega^2_{D-2},$$

with

$$r_H^{D-3} = \frac{16\pi MG_D}{(D-2)\Omega_{D-2}}.$$

Here Ω_n is the volume of a unit n-sphere. For large r, this again determines the Newton potential and therefore the black-hole mass M.

1.2 Schwarzschild solution in Kruskal-Szekeres coordinates

Finally we can bring the black hole metric to the form:

$$ds^2 = \frac{4r_h^3}{r}e^{r_h/r}(-dv^2 + du^2) + r^2 d\Omega_2^2,$$

where $u = (\frac{r}{r_H}-1)^{-1}e^{r/2r_H}cosh(\frac{t}{2r_H}),$

$$v = (\frac{r}{r_H}-1)^{-1}e^{r/2r_H}sinh(\frac{t}{2r_H}).$$

The area of the event horizon is:

$$A = 4\pi r_H^2 = 16\pi(MG)^2.$$

1.3 Reissner-Nordström black hole

The generalization of the Schwarzschild black hole to one with electric charge Q, but no angular momentum, is called the Reissner-Nordström black hole. In four dimensions the metric of a Reissner-Nordström black hole can be written in the form:

$$ds^2 = -\Delta dt^2 + \Delta^{-1}dr^2 + r^2 d\Omega_2^2,$$

where:

$$\Delta = 1 - \frac{2MG}{r} + \frac{Q^2 G}{r^2}.$$

This metric is a solution to Einstein's equations in the presence of an electric field

$$G_{\mu\nu} = R_{\mu\nu} - 1/2Rg_{\mu\nu} = 8\pi GT_{\mu\nu}.$$

1.4 Extremal Reissner-Nordström black hole for D = 4

In the limiting case $r = MG \; or \; M\sqrt{G} = |Q|$ the black hole is called extremal, and it has the maximal charge that is allowed given its mass. The metric of an extremal Reissner-Nordström black hole takes the form:

$$ds^2 = -(1-\frac{r_0}{r})^2 dt^2 + (1-\frac{r_0}{r})^{-2} + r^2 d\Omega_2^2$$

where: $r_0 = MG$.

1.5 Extremal Reissner-Nordström black hole for D = 5

An extremal Reissner-Nordström black hole in D = 5 is of interest in connection with the microscopic derivation of the black-hole entropy. Its metric can be written in a form similar to

$$ds^2 = -\left[(1-\frac{r_0}{r})^2\right]^2 dt^2 + \left[(1-\frac{r_0}{r})^2\right]^{-2} + r^2 d\Omega_3^2.$$

Using this expression, it is easy to see that the horizon at r = 0 has radius r_0, and therefore and its area is:

$$A = \Omega_3 r_0^3 = 2\pi 2 r_0^3.$$

The mass and charge of this black hole are:

$$M = \frac{Q}{\sqrt{G_5}} = \frac{3\pi r_0^2}{4G_5}.$$

2. Black-hole thermodynamics

Classical black holes behave like thermodynamical objects characterized by a temperature and an entropy. The entropy *S* of the system is defined in terms of the number of states as:

$$S(E) = k \ln \Omega(E),$$

where: *k* is Boltzmann's constant.

The temperature *T* of the system is defined in terms of the derivative of the entropy with respect to the energy:

$$\frac{1}{T} = \frac{\partial S}{\partial E}$$

The temperature of the Schwarzschild black hole is:

$$T = \frac{1}{8\pi MG}.$$

For a Schwarzschild black hole the entropy is:

$$S = 4\pi M^2 G.$$

3. Black holes in string theory

This section considers supersymmetric (and hence extremal) black holes that have finite entropy in the supergravity approximation. These include three-charge black holes in five dimensions and four-charge black holes in four dimensions, which can be interpreted as approximations to solutions of toroidally compactified string theory. For this class of compactifications, finite-horizon-area black-hole solutions that are asymptotically flat only exist in the supergravity approximation in four and five dimensions.

4.1 Extremal three-charge black holes for D = 5

The simplest nontrivial example for which the entropy can be calculated involves supersymmetric black holes in five dimensions that carry three different kinds of charges. These can be studied in the context of compactifications of the type **IIB** superstring theory on a five-torus T^5. The analysis is carried out in the approximation that five of the ten dimensions of the **IIB** theory are sufficiently small and the black holes are sufficiently large so that a five-dimensional supergravity analysis can be used.

N=8 supergravity for D=5.

The supergravity theory in question is N = 8 supergravity in five dimensions. This contains a number of one-form and two-form gauge fields.

Three-charge black holes in five dimensions can be obtained by taking Q_1 D1-branes wrapped on an S^1 of radius R inside the T^5, Q_5 D5-branes wrapped on the $T^5 = T^4 \times S^1$, and *n* units of Kaluza-Klein momentum along the same circle. Each of these objects breaks half of the supersymmetry, so altogether 7/8 of the supersymmetry is broken, and one is left with solutions that have four conserved supercharges. Other equivalent string-theoretic constructions of these black-hole solutions are related to the one considered here by U-duality transformations. There are a variety of ways to analyze this system. One of them is in terms of a five-dimensional gauge theory. Since the Q_1 D1-branes are embedded inside the Q_5 D5-branes, this configuration can be described entirely in terms of the $U(Q_5)$ world-volume gauge theory of the D5-branes. In this description a D-string wound on a circle is described by a $U(Q_5)$ instanton that is localized in the other four directions. So, altogether, there are Q_1 such instantons. The Kaluza-Klein momentum can also be described as excitations in this gauge theory. The five-dimensional metric describing this black-hole can be obtained from the ten-dimensional type IIB theory by wrapping the corresponding branes as described above, or it can be constructed directly. In either case, the resulting metric can be written in Einstein frame in the form:

$$ds^2 = -\lambda^{-2/3}dt^2 + \lambda^{1/3}(dr^2 + r^2 d\Omega_3^2)$$

where:

$$\lambda = \prod_{i=1}^{3}\left[1+\left(\frac{r_i}{r}\right)^2\right].$$

This solution describes an extremal three-charge black hole with a vanishing temperature T = 0. The horizon of the black hole is located at r = 0, and its area is:

$$A = 2\pi^2 r_1 r_2 r_3.$$

This vanishes when any of the three charges vanishes, which is the reason that three charges have been considered in the first place. Put differently, one needs to break 7/8 of the super symmetry in order to form a horizon that has finite area in the super gravity approximation, and this requires introducing three different kinds of excitations. The mass of the black hole M to be

$$M = M_1 + M_2 + M_3 \text{ where, } M = \frac{\pi r_i^2}{4G_5}.$$

The fact that the masses are additive in this way is a consequence of the form of the metric. However, this had to be the case, because the BPS condition is satisfied, and the charges are additive. The entropy is:

$$S = \frac{A}{4G_5} = \frac{2\pi g_s l_s^4}{\sqrt{RV}}\sqrt{M_1 M_2 M_3}.$$

In terms of the charges, one obtains the result:

$$S = 2\pi\sqrt{Q_1 Q_5 n}.$$

4.2 Nonextremal three-charge black holes for D = 5

The extremal three-charge black-hole solutions in five dimensions given above have non extremal generalizations, which describe non super symmetric black holes with finite temperature. These black holes are described by the metric

$$ds^2 = -h\,\lambda^{-2/3}dt^2 + \lambda^{1/3}\left(\frac{dr^2}{h} + r^2 d\Omega_3^2\right),$$

where:

$$h = 1 - \frac{r_0^2}{r^2} \text{ and } \lambda = \prod_{i=1}^{3}\left[1+\left(\frac{r_i}{r}\right)^2\right]$$

$$S = \frac{A}{4G_4} = 16\pi G_4\sqrt{M_1 M_2 M_3 M_4}.$$

with $r_i^2 = r_0^2 \sinh^2 \alpha_i$, i = 1, 2, 3.

The mass of this black hole can be read off using the same rules as before resulting in:

$$M = \frac{\pi r_0^2}{8G_5}(\cosh^2\alpha_1 + \cosh^2\alpha_2 + \cosh^2\alpha_3).$$

The area of the horizon is:

$$A = 2\pi^2 r_H^3 = 2\pi^2 r_0^3 \cosh^2\alpha_1 \cosh^2\alpha_2 \cosh^2\alpha_3.$$

The entropy is then given by:

$$S = \frac{A}{4G_5} = \frac{2\pi^2 r_0^3 V_6}{l_p^6} \cosh^2\alpha_1 \cosh^2\alpha_2 \cosh^2\alpha_3.$$

This also allows the entropy to be rewritten in form:

$$S = \frac{A}{4G_5} = 2\pi \prod_{i=1}^{3} (\sqrt{Q_i} + \sqrt{\bar{Q}_i})$$

4.2 Extremal four-charge black holes for D = 4

The construction of super symmetric black holes in four dimensions is quite similar to the five-dimensional case.

$$ds^2 = -\lambda^{-1/2} dt^2 + \lambda^{1/2} \left(dr^2 + r^2 d\Omega_2^2 \right),$$

where

$$\lambda = \prod_{i=1}^{4} \left[1 + \frac{r_i}{r} \right]$$

The mass of the black hole is

$$M = \sum_{i=1}^{4} M_i ,$$

where $M_i = \dfrac{r_i}{4G_4}$.

The area of the horizon, which is located at r = 0, is $A = 4\sqrt{r_1 r_2 r_3 r_4}$.

Putting these facts together, the resulting entropy is:

$$S = \frac{A}{4G_4} = 16\pi G_4 \sqrt{M_1 M_2 M_3 M_4}.$$

4. Conclusions

The existence of black hole entropy indicates the existence of microscopic degrees of freedom which are not present in the usual Einstein theory of gravity. It does not tell us what they are. String theory does provide a microscopic framework for the use of statistical mechanics. In all cases the entropy of the appropriate string system agrees with the Bekenstein-Hawking entropy. This, if nothing else, provides an existence proof for a consistent microscopic theory of black hole entropy.

5. References

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